Answer on question 38667 – Math – Combinatorics

What is the digit at the tens place of (2183)^2156.

Solution

We will use the following binomial formula

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k.$$

We have

$$(21*100+83)^{2156}=\sum_{k=0}^{2156}\binom{2156}{k}(21*100)^{2156-k}83^k=\sum_{k=0}^{2156}\binom{2156}{k}21^{2156-k}83^k100^{2156-k}$$

Only last addend affects on the digit at the tens place. This addend is

$$\binom{2156}{2156}21^{2156-2156}83^{2156}100^{2156-2156}=83^{2156}.$$

Let us consider the last two digits of different powers of 83.

$83^1=83$ $83^2=..89$ $83^3=..87$ $83^4=..21$ $83^5=..43$

$83^6=..69$ $83^7=..27$ $83^8=..41$ $83^9=..03$ $83^{10}=..49$

$83^{11}=..67$ $83^{12}=..61$ $83^{13}=..63$ $83^{14}=..29$ $83^{15}=..07$

$83^{16}=..81$ $83^{17}=..23$ $83^{18}=..09$ $83^{19}=..47$ $83^{20}=..01$

$83^{21}=..83$ …

On the 21th step we can see a pattern. So, we have 20 different variances of the last two digits.

$$2156=107*20+16$$

Therefore, $2183^{2140}=\ldots01$ and $2183^{2156}=\cdots81$ (on the 16th step)

Answer: 8.