Answer on Question#38647 – Math - Other

The language $L = \{a^i b^j c^k \mid i < j < k\}$ is not a context-free.

If L were context free, then the pumping lemma should hold.

Let $z = a^n b^{n+1}\, c^{n+2}$. Given this string and knowing that $|z| \geq n$, we want to define z as $uvwxy$ such that $|vwx| \leq n$, $|vx| \geq 1$. Because $|vwx| \leq n$, there are five possible descriptions of $uvwxy$:

1. vwx is a^p for some $p \leq n, p \geq 1$
2. vwx is $a^p b^q$ for some $p + q \leq n, p + q \geq 1$
3. vwx is b^p for some $p \leq n, p \geq 1$
4. vwx is $b^p c^q$ for some $p + q \leq n, p + q \geq 1$
5. vwx is c^q for some $i \leq n, i \geq 1$

Note that because $|vwx| \leq n$, vwx cannot contain both "a"s and "c". For all of these cases, $uv^i wx^i y$, $i \geq 0$, should be in the language.

In case 1, if $i = 2$ we will be adding an a to the string, making the number of "a"s $n + 1$ and thus the string is not in the language. The same argument holds for *case 3* in which the number of "b"s will be equal to the number of "c"s. A similar argument holds in *case 5*.

In *case 5* if $i = 0$ then the number of "c"s will be less than or equal to the number of "b"s.

In *case 2*, when $i = 2$ either the number of "a"s will be greater than the number of "b"s or the number of "b"s will be greater than the number of "c"s (depending on the distribution of v and x).

In case 4, when $i = 0$ either the number of "b"s will be less than or equal to number of "a"s or the number of "c"s will be less than or equal to the number of "b"s (depending on the distribution of v and x).

Answer: C.