Answer on Question#38460 - Math - Matrix

Initial data:

$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ – Matrix M is a square matrix of dimension 2 because it has 2 eigen values.

$\lambda_1 = 1;\ \lambda_1 = 4$

$v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix};\ v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Solution:

From definition of eigen vectors we can write^

$$\begin{cases} Mv_1 = \lambda_1 v_1 \\ Mv_2 = \lambda_2 v_2 \end{cases}$$

Rewrite this in the extended form:

$$\begin{cases} \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 1\begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 4\begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{cases}$$

From this we have a system of 4 equations with 4 unknowns. We can easy find this unknowns with the substitution method:

$$\begin{cases} a - b = 1 \\ c - d = -1 \\ 2a + b = 8 \\ 2c + d = 4 \end{cases} \quad => \quad \begin{cases} a = 1 + b \\ c = -1 + d \\ 2 + 2b + b = 8 \\ -2 + 2d + d = 4 \end{cases}$$

$$\begin{cases} 3b = 6 \\ 3d = 6 \end{cases} \quad => \quad \begin{cases} b = 2 \\ d = 2 \end{cases}$$

The solution of the system is:

$$\begin{cases} a = 3 \\ c = 1 \\ b = 2 \\ d = 2 \end{cases}$$

So, our matrix M is:

$$M = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$$