Question #38459, Math, Other

The hypotenuse of a right triangles its ends at the points (1,3) and (-4,1). Find the equation of the legs (perpendicular sides) of the triangle.

Solution

The equation of any straight line on a plane can be written in the form

$$Ax + By + C = 0 \quad (1)$$

where $A^2 + B^2 \neq 0$.

By dividing the both sides of the equality (1) by $\sqrt{A^2 + B^2}$ we obtain the equation

$$\frac{A}{\sqrt{A^2+B^2}}x + \frac{B}{\sqrt{A^2+B^2}}y + \frac{C}{\sqrt{A^2+B^2}} = 0,$$

or

$$x\sin\alpha + y\cos\alpha + p = 0, \quad (2)$$

where α is the angle between the line and the opposite direction of the *Ox*-axis.

Let the point (1,3) lie on this line then substituting x=1 and y=3 into the equation (2) we get

$$\sin\alpha + 3\cos\alpha + p = 0. \quad (3)$$

Subtraction of the equation (3) from (1) leads to

$$x\sin\alpha + y\cos\alpha - \sin\alpha - 3\cos\alpha = 0. \quad (4)$$

Thus the equation (4) is the equation of the first leg of the triangle.

The second leg is perpendicular to the first one, so its equation can be written in the form

$$-x\cos\alpha + y\sin\alpha + q = 0. \quad (5)$$

But it passes through the second end of hypotenuse (–4, 1), so we have

$$4\cos\alpha + \sin\alpha + q = 0. \quad (6)$$

By subtracting the equation (5) from the equation (6) we obtain the equation of the second leg of the triangle:

$$x\cos\alpha - y\sin\alpha + 4\cos\alpha + \sin\alpha = 0. \quad (7)$$

So the problem has not an unique solution, because for any angle α the equations (4) and (7) represent a pair of legs of the triangle with given hypotenuse with ends at the points (1,3) and (-4,1).

Particularly by letting $\alpha = 0$ we get from the equations (4) and (7)

$$y = 3 \text{ and } x = -4,$$

that is the pair of legs parallel to the axes.

The value $\alpha = \pi/2$ gives the other pair of legs parallel to the axes:

$$x = 1 \text{ and } y = 1.$$

If $\alpha = \pi/4$ we obtain the next pair

$$x + y - 4 = 0 \text{ and } x - y + 5 = 0.$$

We sketch the discussed pairs of legs in the following figure.