Answer on Question # 38458 – Math – Trigonometry

Find the value of Sin 50°.

Solution

Sine function is infinitely differentiable at a real number. Fifty degrees correspond to

$x = \frac{50}{180}\pi = \frac{5\pi}{18} = \frac{60-10}{180}\pi = \left(\frac{60}{180} - \frac{10}{180}\right)\pi = \frac{\pi}{3} - \frac{\pi}{18}$

The Taylor series

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$

Here

$a = \frac{\pi}{3}, \quad x = \frac{\pi}{3} - \frac{\pi}{18}, \quad x - a = -\frac{\pi}{18}, \quad f(x) = \sin(x), \quad f'(x) = \cos(x), \quad f''(x) = -\sin(x),$

$f^{(3)}(x) = -\cos(x).$

We know the following values

$f(a) = f\left(\frac{\pi}{3}\right) = sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \approx 0.866,$

$f'(a) = f'\left(\frac{\pi}{3}\right) = cos\left(\frac{\pi}{3}\right) = \frac{1}{2} = 0.5,$

$f''(a) = f''\left(\frac{\pi}{3}\right) = -sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \approx -0.866,$

$f^{(3)}(a) = f^{(3)}\left(\frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2} = -0.5,$

$x - a = -\frac{\pi}{18} \approx -0.175,$

$(x-a)^2 = \frac{\pi^2}{18^2} \approx 0.03,$

$(x-a)^3 = -\frac{\pi^3}{18^3} \approx -0.005.$

Collecting terms up to the third power yields

$$Sin\ 50^{\circ} = Sin\left(\frac{5\pi}{18}\right) \approx 0.866 + 0.5 * (-0.175) - \frac{0.866}{2} * 0.03 + \frac{-0.5}{3*2} * (-0.005) \approx 0.766$$

The more terms we take the more accurate answer we obtain.

Calculator shows the following value

$Sin\ 50^{\circ} = Sin\left(\frac{5\pi}{18}\right) \approx 0.766044.$