Answer on Question#38455 – Math - Other

We have $S_n = n^2p$:

$$\frac{n}{2}[2a + (n-1)d] = n^2p$$

Then

$$2a + (n-1)d = 2np \qquad (1)$$

Also, $S_m = m^2p$:

$$\frac{m}{2}[2a + (m-1)d] = m^2p$$

Then

$$2a + (m-1)d = 2mp \qquad (2)$$

$(1) - (2)$:

$$2a + (n-1)d - (2a + (m-1)d) = 2np - 2mp$$

$$nd - d - md + d = 2p(n-m)$$

$$(n-m)d = 2p(n-m)$$

$$d = 2p$$

Put value of d in equation (1):

$$2a + (n-1)2p = 2np$$

$$a + (n-1)p = 2np$$

$$a = np - (n-1)p = np - np + p$$

$$a = p$$

Then

$$S_p = \frac{p}{2}[2a + (p-1)d] = \frac{p}{2}[2p + (p-1)2p] = \frac{p}{2} \cdot 2p \cdot [1 + (p-1)] = \boldsymbol{p^3 = S_p}$$