Answer on Question#38454 - Math - Other

Question: The number of terms of an AP is even, the sum of odd terms is 24, of the even terms is 30, and the last term exceeds the first by 10.5. Find the number of terms and the series.

Solution. Let us denote the first term of the progression by a_1, common difference by d, and number of terms by $2n$ (since we know that this number is even).

Recall that by definition of an arithmetic progression, we have

$$a_n = a_1 + (n-1)d$$

for each n.

Now let us write down the conditions we have for the sums of odd and even terms.

The sum of odd terms is 24:

$$a_1 + a_3 + \cdots + a_{2n-1} = 24.$$

The sum of even terms is 30:

$$a_2 + a_4 + \cdots + a_{2n} = 30.$$

Subtracting the first equality from the second, we obtain

$$a_2 - a_1 + a_4 - a_3 + \cdots + a_{2n} - a_{2n-1} = 30 - 24,$$

which can be written as

$$(a_2 - a_1) + (a_4 - a_3) + \cdots + (a_{2n} - a_{2n-1}) = 6.$$

But the difference between two subsequent terms of an arithmetic progression is equal to the common difference:

$$a_{n+1} - a_n = a_1 + \big((n+1) - 1\big)d - a_1 - (n-1)d = (n - n + 1)d = d.$$

Thus, we have

$$(a_2 - a_1) + (a_4 - a_3) + \cdots + (a_{2n} - a_{2n-1}) = d * \frac{2n}{2} = n * d,$$

and so

$$n * d = 6.$$

We also know that the last term exceeds the first by 10.5:

$$a_{2n} - a_1 = 10.5$$

$$a_1 + (2n-1)d - a_1 = 10.5$$

$$(2n-1)d = 10.5$$

$$2 * n * d - d = 10.5.$$

But we already know that $n * d = 6$, so

$$2 * 6 - d = 10.5$$

$$d = 12 - 10.5 = 1.5,$$

whereas

$$n = \frac{6}{d} = \frac{6}{1.5} = 4,$$

which means that the **number of terms** in the AP is

$$2n = 2 * 4 = 8.$$

Since we already know the common difference, to find the complete **series** it suffices to find a_1.

Recall that the sum of odd terms of the AP is equal to 24:

$$a_1 + a_3 + a_5 + a_7 = 24,$$

or

$$a_1 + (a_1 + 2d) + (a_1 + 4d) + (a_1 + 6d) = 24$$

$$4a_1 + 12d = 24$$

$$4a_1 + 12 * 1.5 = 24$$

$$4a_1 + 18 = 24$$

$$a_1 = \frac{6}{4} = 1.5.$$

Thus, the AP is 1.5; 3; 4.5; 6; 7.5; 9; 10.5; 12.

Answer. The number of terms is 8, and the series is 1.5; 3; 4.5; 6; 7.5; 9; 10.5; 12.