

Answer to Question#38410 – Math – Other

Since P,Q,R are the p-th, q-th, r-th terms of an arithmetic progression with first term a_1 and difference of successive numbers d we have:

$$P = a_1 + (p - 1)d$$

$$Q = a_1 + (q - 1)d$$

$$R = a_1 + (r - 1)d$$

Then

$$\begin{aligned}\sum p(q - r) &= p(q - r) + q(r - p) + r(p - q) \\ &= pq - pr + qr - qp + rp - rq = 0\end{aligned}$$

$$\begin{aligned}\sum p(Q - R) &= p(Q - R) + q(R - P) + r(R - Q) \\ &= p((a_1 + (q - 1)d) - (a_1 + (r - 1)d)) \\ &\quad + q((a_1 + (r - 1)d) - (a_1 + (p - 1)d)) \\ &\quad + r((a_1 + (p - 1)d) - (a_1 + (q - 1)d)) \\ &= pd(q - r) + qd(r - p) + rd(p - q) = d \sum p(q - r) = 0\end{aligned}$$

$$\begin{aligned}\sum P(P - R) &= P(P - R) + Q(Q - P) + R(R - Q) \\ &= (a_1 + (p - 1)d)((a_1 + (p - 1)d) - (a_1 + (r - 1)d)) \\ &\quad + (a_1 + (q - 1)d)((a_1 + (q - 1)d) - (a_1 + (p - 1)d)) \\ &\quad + (a_1 + (r - 1)d)((a_1 + (r - 1)d) - (a_1 + (q - 1)d)) \\ &= (a_1 + (p - 1)d)(p - r)d + (a_1 + (q - 1)d)(q - p)d \\ &\quad + (a_1 + (r - 1)d)(r - q)d \\ &= d(a_1(p - r + q - p + r - q) \\ &\quad + d((p - 1)(p - r) + (q - 1)(q - p) + (r - 1)(r - q))) \\ &= d^2(p^2 + q^2 + r^2 - pr - pq - rq) \neq 0\end{aligned}$$

ANSWER: A, B