

### Answer on Question #38379 - Math - Differential Calculus

$$x \frac{dy}{dx} + 3y = 6x$$

Let's divide this equation by  $x$ :

$$\frac{dy}{dx} + 3 \frac{y}{x} = 6$$

Firstly let's find general solution of homogeneous equation

$$\frac{dy}{dx} + 3 \frac{y}{x} = 0$$

Let's solve this equation using separation of variables.

$$\frac{dy}{dx} = - \frac{3y}{x}$$

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Integrating this equation:

$$\ln y = -3 \ln x + \ln c$$

$$y = \frac{c}{x^3}$$

To solve non-homogeneous initial equation let's take constant  $c$  as function of  $x$ :  $c(x)$ . Substituting  $y = \frac{c(x)}{x^3}$  this into the equation we get:

$$x \cdot \frac{c' \cdot x^3 - c \cdot 3x^2}{x^6} + \frac{3c}{x^3} = 6x$$

$$\frac{c'}{x^2} = 6x$$

$$c' = 6x^3$$

$$c = \frac{3}{2}x^4 + c_1$$

Thus general solution of initial equation is

$$y(x) = \frac{c(x)}{x^3} = \frac{3}{2}x + \frac{c}{x^3}$$

where  $c \in \mathbb{R}$  is an arbitrary constant.