Question #38293, Math, Calculus

The population (P) of an island y years after colonisation is given by the function

  P =250/1+4e^(-0.01y)

What was the initial population of the island?

How long did it take before the island had a population of 150?

After how many years was the population growing the fastest?

Sketch the function.

Give a possible explanation for the shape of the curve.

Solution

Since the population must grow then the parentheses were omitted, i. e.

$$P = \frac{250}{1 + 4e^{-0.01y}} = \frac{250e^{0.01y}}{e^{0.01y} + 4}. \tag{1}$$

We get the size of initial population of the island by substituting 0 for *x* in the formula (1)

$$P(0) = 250/(1 + 4) = 50.$$

The substitution 150 for *P* in the above formula gives us the equation

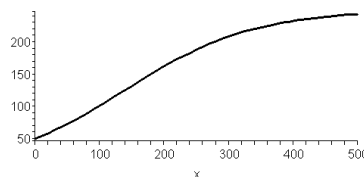$$\frac{250e^{0.01y}}{e^{0.01y} + 4} = 150,$$

which has the solution *y* = 100ln(6) = 179.18. Hence it took almost 180 years before the island had a population of 150.

The derivative $\frac{dP}{dy}$ is equal to

$$P' = \frac{10}{(e^{0.02y} + 4e^{0.01y})}.$$

It is a decreasing function of the number of passing years *y*, so the population growing was the fastest at the initial state (for *y* = 0).

The function graph is sketched in the Figure.

In my opinion the shape of the curve reflects the fact that population increasing is limited by water and food shortage increasing.