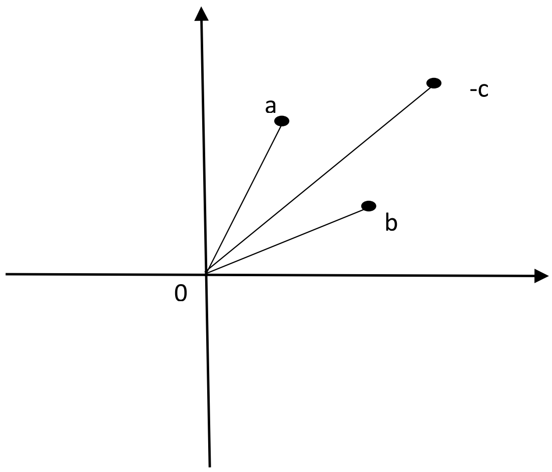
Answer on question 38139 – Math – Complex Analysis

if SinA+sinB+sinC=0,cosA+cosB+cosC=0 then prove that A-B=2pi/3, B-C=2pi/3, C-A=2pi/3

let us consider three complex numbers $a = cosA + i\ sinA, b = \cos B + i \sin B\ , c = \cos C + i \sin C$. Moreover, $|a| = |b| = |c| = 1$.

$$a + b + c = (\cos A + \cos B + \cos C) + i(\sin A + \sin B + \sin C) = 0$$

Therefore, $a + b = -c$. Look at the graph



-c is a diagonal of the parallelogram 0a-cb. We know that $|a| = |b| = |c| = 1$, therefore, we obtain that triangles 0a-c and 0-cb are equilateral triangles. So the angle a0b=2*60=120 degrees or equals 2pi/3. This is the angle A-B.

Similarly, we can find that B-C=C-A=2pi/3.

QED.