Answer on Question#38070 – Math - Calculus

The system becomes

$$(yz, xz, xy) = \lambda(2x, 2y, 0) + \mu(1, 0, -1), \quad x^2 + y^2 - 1 = 0, \quad x - z = 0$$

or

$$\begin{cases} yz = 2\lambda x + \mu \\ xz = 2\lambda y \\ xy = -\mu \\ x^2 + y^2 - 1 = 0 \\ x - z = 0 \end{cases}$$

Expressing μ from the third and z from the last equation and substituting, we get

$$\begin{cases} z = x \\ \mu = -xy \\ 2xy = 2\lambda x \\ x^2 = 2\lambda y \\ x^2 + y^2 - 1 = 0 \end{cases}$$

Next we want to eliminate λ but we have to divide e.g. by x in the third equation. This is only allowed if $x \neq 0$, so we have to treat cases:

1. $x = 0$ which yields $\lambda = 0$ and $y = \pm 1$, hence two points $(0, \pm 1, 0)$. The value of f at both points is 0.
2. $x \neq 0$ and we get $\lambda = y$, hence $x^2 = 2y^2$ from the 4th equation. Substitution into the last one yields $3y^2 = 1$ or $y = \pm\frac{1}{\sqrt{3}}$ and $z = x = \pm\sqrt{\frac{2}{3}}$. Here the signs for x and y can be chosen independently and $z = x$ from the first equation. Hence we obtain 4 solutions:

$$\left(\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right), \left(\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right), \left(-\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\right), \left(-\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\right)$$

leading to the values of f equal to $\pm\frac{2}{3\sqrt{3}}$. The required minimum is the minimum of the values obtained which is $-\frac{2}{3\sqrt{3}}$.