**Answer on Question#38069 – Math - Calculus**

**a.** We have

$$f(x,y)=x^2y^3-y^4,\ \ (2,1),\ \ \theta=\pi/4$$

Find $\vec{\nabla} f(x,y)$:

$$\vec{\nabla} f(x,y)=\langle\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\rangle=\langle 2xy^3,3x^2y^2-4y^3\rangle$$

So

$$\vec{\nabla} f(2,1)=\langle 2\cdot 2\cdot 1^3,3\cdot 2^2\cdot 1^2-4\cdot 1^3\rangle=\langle 4,8\rangle$$

Find $\vec{u}$:

$$\vec{u}=\langle\cos\theta\,,\sin\theta\rangle=\langle\sqrt{2}/2\,,\sqrt{2}/2\rangle$$

Then the directional derivative of $f(x,y)$:

$$D_{\vec{u}}f=\vec{\nabla} f\cdot\vec{u}=\langle 4,8\rangle\cdot\langle\sqrt{2}/2\,,\sqrt{2}/2\rangle=2\sqrt{2}+4\sqrt{2}=6\sqrt{2}$$

**b.** We have

$$f(x,y,z)=\sqrt{xyz},(3,2,6),\vec{v}=\langle 1,2,2\rangle$$

Find $\vec{\nabla} f(x,y,z)$:

$$\vec{\nabla} f(x,y,z)=\langle\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\rangle=\frac{\langle yz,xz,xy\rangle}{2\sqrt{xyz}}$$

So

$$\vec{\nabla} f(3,2,6)=\frac{\langle 12,18,6\rangle}{2\sqrt{36}}=\frac{\langle 12,18,6\rangle}{12}=\langle 1,\frac{3}{2},\frac{1}{2}\rangle$$

Find $\vec{u}$:

$$\vec{u}=\frac{\vec{v}}{|\vec{v}|}=\frac{\langle 1,2,2\rangle}{\sqrt{1+4+4}}=\frac{\langle 1,2,2\rangle}{3}=\langle\frac{1}{3},\frac{2}{3},\frac{2}{3}\rangle$$

Then the directional derivative of $f(x,y,z)$:

$$D_{\vec{u}}f=\vec{\nabla} f\cdot\vec{u}=\langle 1,\frac{3}{2},\frac{1}{2}\rangle\cdot\langle\frac{1}{3},\frac{2}{3},\frac{2}{3}\rangle=\frac{1}{3}+1+\frac{1}{3}=\frac{5}{3}$$

**Answer:**

- **a.** $D_{\vec{u}}f=6\sqrt{2}$;
- **b.** $D_{\vec{u}}f=5/3$.