Answer on Question #38030, Math, Statistics and Probability

Assignment

Heights of women have a bell shaped dist. mean of 160cm sd 0f 5cm using chebyshev's theorems, what are the minimum and maximum heights that are within 3 standard deviations of the mean.

Solution

We apply one of Chebyshev`s theorem, namely Chebyshev`s inequality

$$\boldsymbol{Pr(|\xi - E\xi| < \varepsilon) \geq \frac{D\xi}{\varepsilon^2}}.$$

By the statement of the problem, $E\xi = 160$, $D\xi = \sigma^2 = 0.5^2$, $\varepsilon = 3\sigma$, so

$$Pr(|\xi - 160| < 3\sigma) \geq \frac{0.5^2}{9 * 0.5^2} = \frac{1}{9} \approx 0.11$$

Thus, according to Chebyshev`s inequality we state that with probability 0.11 heights of women range from $E\xi - 3\sigma = 160 - 3 * 0.5 = 158.5$ to $E\xi + 3\sigma = 160 + 3 * 0.5 = 161.5$

Since we know exact distribution, we can propose more exact value of this probability, i.e.

$Pr(|\xi - E\xi| < 3\sigma) = Pr\left(\frac{|\xi - E\xi|}{\sigma} < 3\right) = Pr(|\eta| < 3) = Pr(-3 < \eta < 3) =$

$= \Pr(\eta < 3) - \Pr(\eta < -3) = F(3) - F(-3) = 2 * 0.49865 = 09973$

(this value is more exact than one from Chebyshev`s inequality), where *F(x)* is cumulative distribution function of the standard normally distributed variable $\eta = \frac{\xi - E\xi}{\sigma}$.

Answer: 158.5; 161.5