Answer on Question #37865, Math, Other.

During a typical one-hour lunch period 28 people go to a local deli. During a randomly chosen 15 minute period, what is the probability that:
a. Only 6 people go to the deli?
b. More than 12 people go to the deli?
c. At most 10 people go to the deli?
d. No more than 5 people go to the deli?

Solution.

It's obvious that the process depicted in the problem statement is Poisson process: we have stohastic events(each person who goes to a local deli), which occur with constant known average rate $\lambda = 28$ and independently of time of the last event. The parameter of time for a,b,c,d is t=15/60=1/4.

Let's look at concrete cases:

a)exactly 6 people(events) in 15 minutes:

$$P(\tau, k) = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!};$$

$P\left(\frac{1}{4}, 6\right) = \frac{\left(\frac{1}{4}\cdot 28\right)^6 e^{-\frac{1}{4}\cdot 28}}{6!}$ - probability that 6 people go to the deli.

b)more than 12 people (in this case it is needed to find the sum of probabilities that nobody will come, one person will come, two people, …, 12 people will come and substract this sum from 1):

$$P(X > 12) = 1 - e^{-28\cdot\frac{1}{4}} \sum_{k=0}^{12} \left(\frac{(7)^k}{k!}\right).$$

c)at most 10 people (in this case it is needed to find the sum of probabilities that nobody will come, one person will come, two people, …, 10 people will come):

$$P(X \le 10) = e^{-28\cdot\frac{1}{4}} \sum_{k=0}^{10} \left(\frac{(7)^k}{k!}\right).$$

d) no more than 5 people (it is the same to problem c) up to the parameter k. It is 5, in previous problem it was 10):

$$P(X \le 5) = e^{-28\cdot\frac{1}{4}} \sum_{k=0}^{5} \left(\frac{(7)^k}{k!}\right).$$