Question #37830, Math, Calculus

Suppose that we don't have a formula for g but we know that g(2)=-4 and g'(x)= sqt(x^2+5)

-use a linear approximation to estimate g(1.95) and g(2.05)

-solve

Solution

The value of the derivative of any continuous function, $g'(x_0)$, can be used to estimate values of $f(x)$ for x-values near x_0. Recall the difference quotient

$$\frac{g(x+h)-g(x)}{h}.$$

If delta notation is used, the difference quotient becomes

$$\frac{g(x+\Delta x)-g(x)}{\Delta x}.$$

We can then express the derivative as

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}.$$

For values of Δx close to 0, we have the approximation

$$\frac{dy}{dx}\approx\frac{\Delta y}{\Delta x}\ or\ g'(x)\approx\frac{\Delta y}{\Delta x}$$

Multiplying both sides of the second expression by Δx gives us

$$\Delta y\approx g'(x)\cdot\Delta x. \qquad (1)$$

This approximation is the linear one because we use the y-values on the tangent line to estimate function values for small values of Δx.

Since

$$\Delta y = g(x + \Delta x) - g(x),$$

Then, we get from the formula (1)

$$g(x + \Delta x) \approx g(x) + g'(x)\cdot\Delta x. \qquad (2)$$

From (2) for $x = 2$ and $\Delta x = 1.95 - 2 = -0.05$ we find

$$g(1.95) \approx g(2) + g'(2)\cdot(-0.05) = -4 + \sqrt{2^2+5}\cdot(-0.05) = -4.15.$$

And similarly for $x = 2.05$ and $\Delta x = 2.05 - 2 = 0.05$ we obtain

$$g(2.05) \approx g(2) + g'(2)\cdot 0.05 = -4 + \sqrt{2^2+5}\cdot 0.05 = -3.85.$$

Answers:

$g(1.95) \approx -4.15$; $g(2.05) \approx -3.85$.