Answer on Question #37770 – Math - Integral Calculus

We have

$$\int \big(f(x) - g(x)\big)dx = f(x)x^3 - \int 3x^2 \sin x \, dx \qquad (1)$$

Use this property of antiderivative to find $f(x) - g(x)$:

$$\int u dv = uv - \int v du$$

Then $udv = \big(f(x) - g(x)\big)dx, \;\; vu = f(x)x^3, \;\; vdu = 3x^2 \sin x \, dx = \sin x \, dx^3.$

We can see that $v = f(x) = \sin x \, , u = x^3$

Substitute it into (1):

$$\int \big(\sin x - g(x)\big)dx = x^3 \sin x - \int 3x^2 \sin x \, dx$$

Use this property to find $g(x)$:

$$\frac{d}{dx}\left(\int F(x)dx\right) = F(x)$$

$$\frac{d}{dx}(\ldots) \;\Big|\; \int \big(\sin x - g(x)\big)dx = x^3 \sin x - \int 3x^2 \sin x \, dx$$

$$\sin x - g(x) = x^3 \cos x + 3x^2 \sin x - 3x^2 \sin x$$

$$\sin x - x^3 \cos x = g(x)$$

So

$$f(x) - g(x) = \sin x - \sin x + x^3 \cos x = x^3 \cos x$$

Answer: $f(x) - g(x) = x^3 \cos x$.