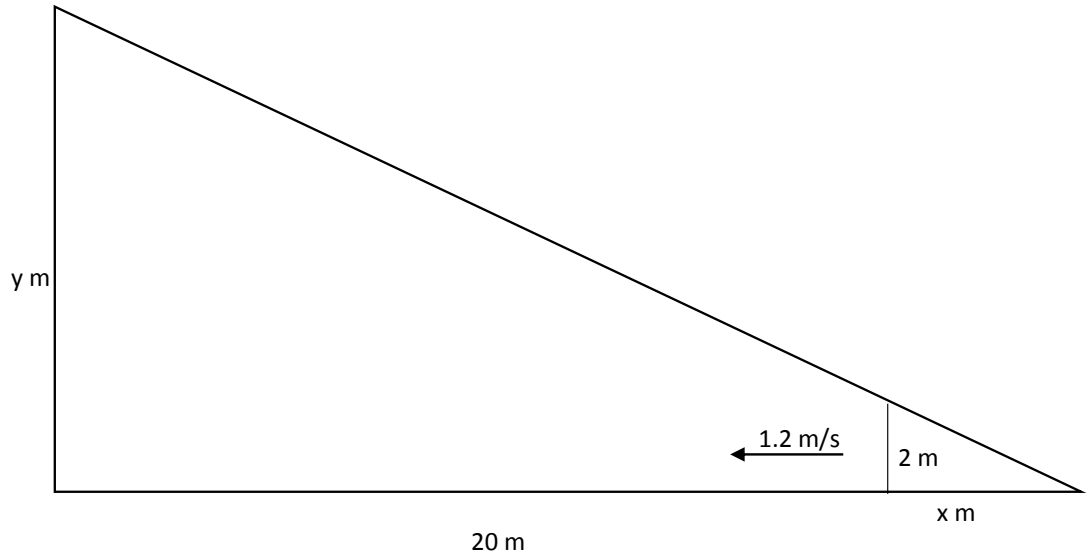
Answer on question 37740 – Math – Calculus

A spotlight on the ground is shining on a wall 20 m away. If a woman 2 m tall walks from the spotlight toward the building at a speed of 1.2 m/s, how fast is the length of her shadow on the building decreasing when she is 4 m from the building?

Solution



Let x be the distance between the spotlight and the man and y be the length of his shadow on the building (both in meters). Then $\frac{2}{y} = \frac{x}{20}$ and hence $xy = 40$. Take derivative on both sides of $xy = 40$ with respect to t:

$$x\left(\frac{dy}{dt}\right) + \left(\frac{dx}{dt}\right)y = 0 \;\Rightarrow \frac{dy}{dt} = -\frac{y}{x}\left(\frac{dx}{dt}\right)$$

Plug in $\frac{dx}{dt} = 1.2, x = 20 - 4 = 16$ and $y = \frac{40}{16} = 2.5$ and we obtain

$$\frac{dy}{dt} = -\frac{2.5}{16} * 1.2 = -0.1875.$$

So his shadow is decreasing at a rate of 0.1875 m/s.

Answer: 0.1875 m/s.