

Answer on Question#37727 – Math - Integral Calculus

Find $\int \sin^3 x dx$

Solution:

Writing $\sin^3 x = \sin x(1 - \cos^2 x)$

$$\begin{aligned}\int \sin^3 x dx &= \int \sin x(1 - \cos^2 x) dx = \int \sin x dx - \int \sin x \cos^2 x dx = \\ &= -\cos x - \int \sin x \cos^2 x dx\end{aligned}$$

Let $u = \cos x$, the $\frac{du}{dx} = -\sin x$, $du = -\sin x dx$ and

$$\int \sin x \cos^2 x dx = -\int u^2 du = -\frac{1}{3}u^3 + C = -\frac{1}{3}\cos^3 x + C$$

So

$$\int \sin^3 x dx = -\cos x + \frac{1}{3}\cos^3 x + C$$