

Answer on Question#31412 – Math – Trigonometry

Condition of the problem:

What is the exact value of $4 \cos(540^\circ) + 3 \operatorname{tg}(-405^\circ)$?

Solution:

It is known that

$$540^\circ = 3\pi,$$
$$\cos(540^\circ) = \cos(3\pi) = -1.$$

It is known that

$$405^\circ = 360^\circ + 45^\circ,$$

Using next formulas to calculate the $\operatorname{tg}(-405^\circ)$:

$$\operatorname{tg}(x) = \frac{\sin(x)}{\cos(x)},$$

$$\sin(-x) = -\sin(x), \quad \cos(-x) = \cos(x),$$

$$\sin(x + y) = \sin(x) \cos(y) + \sin(y) \cos(x),$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y),$$

$$\sin(360^\circ) = 0, \quad \cos(360^\circ) = 1,$$

$$\sin(45^\circ) = \frac{\sqrt{2}}{2}, \quad \cos(45^\circ) = \frac{\sqrt{2}}{2}.$$

$$\begin{aligned} \operatorname{tg}(-405^\circ) &= \frac{\sin(-405^\circ)}{\cos(-405^\circ)} = \frac{-\sin(405^\circ)}{\cos(405^\circ)} = \frac{-\sin(360^\circ + 45^\circ)}{\cos(360^\circ + 45^\circ)} = \\ &= \frac{-\sin(360^\circ) \cos(45^\circ) - \sin(45^\circ) \cos(360^\circ)}{\cos(360^\circ) \cos(45^\circ) - \sin(45^\circ) \sin(360^\circ)} = \frac{-\sin(45^\circ)}{\cos(45^\circ)} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1. \end{aligned}$$

Conclusion:

$$4 \cos(540^\circ) + 3 \operatorname{tg}(-405^\circ) = 4 \cdot (-1) + 3 \cdot (-1) = -7.$$

Answer: -7 .