Answer on Question 37639 – Math - Calculus

For the function f(x), f(x) = x + a + integral from 0 to 1 of {f(t)}^2 dt

is true. What is the maximum value for the constant a?

a) -3/4

b) -1/2

c) -1/3

d) 1/3

e) ½

Solution

The function is given by the equation

$$f(x) = x + a + \int_0^1 f^2(t)dt. \quad (1)$$

Since $\int_0^1 f^2(t)dt$ is a nonnegative real number, then $b = a + \int_0^1 f^2(t)dt$ is also some real number and $b \geq a$. Then from (1) we obtain

$$f(x) = x + b, \quad (2)$$

or $f(t) = t + b$, so

$$\int_0^1 f^2(t)dt = \int_0^1 (t+b)^2 dt = \frac{(t+b)^3}{3}\Big|_0^1 = b^2 + b + \frac{1}{3}. \quad (3)$$

From (1) - (3) we get the following equality:

$$x + b = x + a + b^2 + b + 1/3.$$

Hence,

$$a = -b^2 - 1/3.$$

Since $b \geq a$ we get all possible values of parameter b from the inequality

$$b^2 + b + 1/3 \geq 0.$$

Its solution is all real numbers, so we must find the maximum value for the function $a = -b^2 - 1/3$ for all real b. Since $a \leq -1/3$ then a takes the maximum value for $b = 0$ and it is equal to $-1/3$.

Answer

c) -1/3