Answer on Question #37620, Math, Differential Calculus | Equations

The length of the curve if it is given in form $y=f(x)$ from point x_0 to x_1 is $S=\int_{x_0}^{x_1}\sqrt{1+(f_x)^2}\,dx$. For $f(x)=\ln sec\, x=\ln\frac{1}{\cos x}$:

$$S=\int_0^{\frac{\pi}{3}}\sqrt{1+\frac{\sin^2 x}{\cos^2 x}}\,dx=\int_0^{\frac{\pi}{3}}\sqrt{1+tg^2 x}\,dx=\int_0^{\frac{\pi}{3}}\frac{dx}{\cos x}=-\ln\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)+\ln\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\Big|_0^{\frac{\pi}{3}}=$$

$$-\ln\left(\frac{\sqrt{3}-1}{2}\right)+\ln\left(\frac{\sqrt{3}+1}{2}\right)=\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)=\ln\left(\sqrt{3}+2\right) .$$