Answer on Question #37548 – Math - Differential Calculus

Let p, t, and r represent the principal, time, and rate of interest respectively. It is given that the principal increases continuously at the rate of r % per year. So we have

$$\frac{dp}{dt} = p\left(\frac{r}{100}\right)$$

$$\frac{dp}{p} = \frac{r}{100}dt$$

Integrate both sides

$$\int \frac{dp}{p} = \int \frac{r}{100}dt$$

$$\ln p = \frac{rt}{100} + const = \frac{rt}{100} + k$$

$$p = e^{\frac{rt}{100}+k}$$

For $t = 0, p = 100$:

$$100 = e^{0+k} = e^k$$

For $t = 10$, we have $p = 2 \cdot 100 = 200$:

$$200 = e^{\frac{r}{10}+k}$$

$$200 = e^{\frac{r}{10}} \cdot e^k = e^{\frac{r}{10}} \cdot 100$$

$$\frac{r}{10} = \ln 2$$

$$r = 0.6931 \cdot 10 = 6.931$$

The value of r is 6.93%.

Consider the second case. Let p and t be the principal and time respectively. It is given that the principal increases continuously at the rate 5% per year. Then we have

$$\frac{dp}{dt} = p\left(\frac{5}{100}\right)$$

$$\int \frac{dp}{p} = \int \frac{1}{20}dt$$

$$\ln p = \frac{t}{20} + k$$

$$p = e^{\frac{t}{20}+k}$$

For $t = 0, p = 1000$:

$$1000 = e^{0+k} = e^k$$

If $t = 10$, then:

$$p = e^{\frac{1}{2}+k}$$

$$p = e^{1/2} \cdot e^k = e^{\frac{1}{2}} \cdot 1000 = 1.648 \cdot 1000 = 1648$$

After 10 years the amount will worth s 1648.