Answer on Question #37525-Math-Calculus

A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point (4,2), its x-coordinate increases at a rate of 3 cm/s. How fast is the distance from the particle to the origin changing at this instant?

Solution

Let s be the distance of the particle from the origin.

We have $s^2 = x^2 + y^2 = x^2 + x$, since $y = \sqrt{x}$.

Differentiating with respect to t:

$$2s\frac{ds}{dt} = (2x + 1)\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{2x+1}{2\sqrt{x^2+x}}\frac{dx}{dt}$$

We are given that $\frac{dx}{dt} = 3$ at (4,2).

Thus, when $x = 4$,

$$\frac{ds}{dt} = \frac{2 \cdot 4 + 1}{2\sqrt{4^2+4}} \cdot 3 = \frac{27}{4\sqrt{5}} = \frac{27\sqrt{5}}{20}$$

So at that instant the distance from the particle to the origin is increasing at $\frac{27\sqrt{5}}{20}$ cm/s.

Answer

$\frac{27\sqrt{5}}{20}$ cm/s