Answer on Question #37417 – Math - Differential Calculus

Taylor's theorem.

Suppose $f(x)$ is differentiable at the point $x = b$. Then this function can be expanded into Taylor series at the point $x = b$ and expressed by the following formula

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(b)}{n!}(x-b)^n$$

At first, we should find derivatives of the sine function and its value at the point 0.

$$f'(x) = \cos x \qquad f'(0) = 1$$

$$f''(x) = -\sin x \qquad f''(0) = 0$$

$$f'''(x) = -\cos x \qquad f'''(0) = -1$$

$$f^{(4)}(x) = \sin x \qquad f^{(4)}(0) = 0$$

$$f^{(5)}(x) = \cos x \qquad f^{(5)}(0) = 1$$

Substituting this into general formula we get

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

Or in compact sigma notation that is equal to

$$\sin x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$