

Question#37497 - Mathematics - Differential Calculus

If $y = e^{ax} \cos^3 x \sin^2 x$ find $\frac{dy}{dx}$

Solution:

Using

Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x) \cdot h(x)) = f(x) \cdot g(x) \cdot \frac{d}{dx}(h(x)) + f(x) \cdot h(x) \cdot \frac{d}{dx}(g(x)) + h(x) \cdot g(x) \cdot \frac{d}{dx}(f(x))$$

Power rule

$$\frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

The chain rule

If $h(x) = f(g(x))$, then

$$\frac{dh}{dx} = \frac{dh}{dg} \cdot \frac{dg}{dx}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{dy}{dx} = e^{ax} \cos^3 x \frac{d}{dx}(\sin^2 x) + e^{ax} \sin^2 x \frac{d}{dx}(\cos^3 x) + \cos^3 x \sin^2 x \frac{d}{dx}(e^{ax}) =$$

$$= e^{ax} \cdot \cos^3 x \cdot 2 \sin x \cdot \cos x + e^{ax} \cdot \sin^2 x \cdot 3 \cos^2 x \cdot (-\sin x) + \cos^3 x \cdot \sin^2 x \cdot a \cdot e^{ax}$$

Answer:

$$\frac{dy}{dx} = e^{ax} \cdot \cos^2 x \sin x (2 \sin x \cdot \cos^2 x - 3 \sin^2 x + a \sin x \cdot \cos x)$$