Answer on Question# 37458 - Math - Graph Theory

Let G be a graph with 100 vertices numbered 1 to 100. Two vertices i and j are adjacent if |i-j|=8 or |i-j|=12. The number of connected components in G is
a)8
b)12
c)25
d)4

Answer:

Consider the case $|i - j| = 8$:

then we could partition the graph as 8 sets , the vertices in each set are connected:

$\quad$ s1 { 1,9,17,....} , i.e all the vertices of form $8x + 1$ are connected to, x >= 0

$\quad$ s2 {2,10,18,....} i.e all the vertices of form $8x + 2$ are connected , x >= 0

$\quad$ s3 {3,11,19,....} i.e all the vertices of form $8x + 3$ are connected , x >= 0

$\quad$.

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$\quad$ s7 { 7,15,.....} i.e all the vertices of form $8x + 7$ are connected , x >= 0

$\quad$ s8 { 8,16,.....} i.e all the vertices of form $8x$ are connected , x >= 1.

Now consider the case $|i - j| = 12$:

$\quad\quad$ observe that 1 and 13 are connected, so $s1, s5$ together form a component

$\quad\quad$ observe that 2 and 14 are connected, so $s2, s6$ together form a component

$\quad\quad$ observe that 3 and 15 are connected, so $s3, s7$ together form a component

$\quad\quad$ observe that 4 and 16 are connected, so $s4, s8$ together form a component

$\Rightarrow$ so total number of components are 4

Answer: d) 4.