Answer on Question#37428 - Math - Statistics and Probability

Let's denote possible events in such a way:

A – the chosen coin is 2 heads

B – the chosen coin is normal

H – the head turns up each time

We need to find the probability that the chosen coin is two-headed given that heads turned up each time. SO we need to find conditional probability of event A given event H.

$$P(A|H)$$

By Bayes' theorem:

$$P(A|H) = \frac{P(H|A)P(A)}{P(H)}$$

By law of total probability

$$P(H) = (H|A)P(A) + P(H|B)P(B)$$

Substituting this we get:

$$P(A|H) = \frac{P(H|A)P(A)}{P(H|A)P(A) + P(H|B)P(B)}$$

Since tosses of 2 heads coin produces only heads,

$$P(H|A) = 1$$

Probability that a head will appear 4 times in case of normal coin:

$$P(H|B) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

There are 3 coin totally, and 2 of them are normal, thus

$$P(A) = \frac{1}{3}$$

$$P(B) = \frac{2}{3}$$

Substituting this numerical values into the formula we get:

$$P(A|H) = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + \frac{1}{16} \cdot \frac{2}{3}} = \frac{8}{9}$$

Thus answer is $\frac{8}{9}$.