Question 37380: Find the n^{th} derivative of

(I.) $\cos mx$
(II.) $\cos 2x$
(III.) $\cos^2 x$

Solution.

(I.) $f(x) = \cos mx$

$$f'(x) = -m * \sin mx = m * \cos\left(mx - \frac{\pi}{2}\right)$$
$$f''(x) = -m^2 * \cos mx = m^2 * \cos(mx - \pi)$$
$$f'''(x) = m^3 * \sin mx = m^3 * \cos\left(mx - \frac{3\pi}{2}\right)$$

We can now write a formula for the general case (for an arbitrary $n \in \mathbb{N}$):

$$f^{(n)}(x) = m^n * \cos\left(mx - \frac{n * \pi}{2}\right).$$

(II.) $g(x) = \cos 2x$

This is a special case of $\cos mx$ ($m = 2$). Applying the same logic as above, we have

$$g^{(n)}(x) = 2^n * \cos\left(2x - \frac{n * \pi}{2}\right).$$

(III.) $h(x) = \cos^2 x = \frac{1}{2} * (1 + \cos 2x)$.

$$h'(x) = \frac{1}{2} * 2 * \sin 2x = \frac{2}{2} * \cos\left(2x - \frac{\pi}{2}\right)$$
$$h''(x) = -2 * \cos 2x = \frac{2^2}{2} * \cos(2x - \pi)$$
$$h'''(x) = 4 * \sin 2x = \frac{2^3}{2} * \cos\left(2x - \frac{3\pi}{2}\right)$$

Thus, for every $n \in \mathbb{N}$:

$$h^{(n)}(x) = 2^{n-1} * \cos\left(2x - \frac{n * \pi}{2}\right).$$

Answer.

(I.) $\cos^{(\mathrm{n})} mx = m^n * \cos\left(mx - \frac{n*\pi}{2}\right)$.
(II.) $\cos^{(\mathrm{n})} 2x = 2^n * \cos\left(2x - \frac{n*\pi}{2}\right)$.
(III.) $(\cos^2 x)^{(\mathrm{n})} = 2^{n-1} * \cos\left(2x - \frac{n*\pi}{2}\right)$.