Find

$$\frac{dy}{dx}$$

if

$$y(x) = e^{ax}\cos^3(x)\sin^2(x).$$

Solution:

We have

$$\frac{dy}{dx} = \left(e^{ax}\cos^3(x)\sin^2(x)\right)' = \left(e^{ax}\right)'\cos^3(x)\sin^2(x) + e^{ax}\left(\cos^3(x)\right)'\sin^2(x) +$$

$$+e^{ax}\cos^3(x)\left(\sin^2(x)\right)' = \left(ae^{ax}\right)\cdot\cos^3(x)\sin^2(x) + e^{ax}\cdot\left(-3\cos^2(x)\cdot\sin(x)\right)\cdot\sin^2(x) +$$

$$+e^{ax}\cos^3(x)\cdot\left(2\sin(x)\cdot\cos(x)\right).$$

We'll use that

$$\frac{1}{2}\sin(2x) = \sin(x)\cos(x)\,;$$

$$\sin^2(x) = 1 - \cos^2(x).$$

So

$$\frac{dy}{dx} = \frac{1}{2}e^{ax}\sin(2x)\cos(x)\left(a\cos(x)\sin(x) - 3\sin^2(x) + 2\cos^2(x)\right) =$$

$$= \frac{1}{2}e^{ax}\sin(2x)\cos(x)\left(\frac{a}{2}\sin(2x) - 3\left(1-\cos^2(x)\right) + 2\cos^2(x)\right) =$$

$$= \frac{1}{2}e^{ax}\sin(2x)\cos(x)\left(\frac{a}{2}\sin(2x) + 5\cos^2(x) - 3\right).$$

Answer:

$$\frac{dy}{dx} = \frac{1}{2}e^{ax}\sin(2x)\cos(x)\left(\frac{a}{2}\sin(2x) + 5\cos^2(x) - 3\right)$$