

The gradient of the curve $y - xy + 2px + 3qy = 0$ at the point (3,2) is $-2/3$. The value of p and q are?

Solution:

We have

$$y - xy + 2px + 3qy = 0.$$

Thus

$$\frac{d}{dx}(y - xy + 2px + 3qy) = 0,$$

$$\frac{dy}{dx} - x \frac{dy}{dx} - y + 2p + 3q \frac{dy}{dx} = 0.$$

We have two conditions

$$y|_{x=3} = 2$$

and

$$\frac{dy}{dx} \Big|_{\substack{x=3 \\ y=2}} = -\frac{2}{3}.$$

So we get system of two equations

$$\begin{cases} 2 - 3 \cdot 2 + 2p \cdot 3 + 3q \cdot 2 = 0, \\ -\frac{2}{3} - 3 \cdot \left(-\frac{2}{3}\right) - 2 + 2p + 3q \cdot \left(-\frac{2}{3}\right) = 0, \end{cases}$$

$$\begin{cases} 3p + 3q = 2, \\ p - q = \frac{1}{3}, \end{cases} \Rightarrow \begin{cases} p + q = \frac{2}{3}, \\ p - q = \frac{1}{3}, \end{cases} \Rightarrow \begin{cases} 2q = \frac{1}{3}, \\ 2p = 1, \end{cases} \Rightarrow \begin{cases} q = \frac{1}{6}, \\ p = \frac{1}{2}. \end{cases}$$

Answer:

$$q = \frac{1}{6}$$

$$p = \frac{1}{2}$$