Answer on Question #37128 – Math – Differential Geometry

Question.

Find the tangent line of the curve at the points $s = +1, -1, 0$

$$r(s) = (s^2 - 1, s(s^2 - 1), 0)$$

Solution.

The tangent line of the curve

$$x = x(s)$$

$$y = y(s)$$

$$z = z(s)$$

in the point s_0 is the line

$$\frac{x - x_{0i}}{x'_{0i}} = \frac{y - y_{0i}}{y'_{0i}} = \frac{z - z_{0i}}{z'_{0i}}, i = 1,2,3$$

In our case, we have:

$$x(s) = s^2 - 1$$

$$y(s) = s(s^2 - 1)$$

$$z(s) = 0$$

$$x_{01} = s^2 - 1_{s=1} = 0, y_{01} = 0, z_{01} = 0$$

$$x_{02} = s^2 - 1_{s=-1} = 0, y_{02} = 0, z_{02} = 0$$

$$x_{03} = s^2 - 1_{s=0} = -1, y_{03} = 0, z_{03} = 0$$

$$x'(s) = 2s, y'(s) = 3s^2 - 1, z'(s) = 0$$

$$x'_{01} = 2s_{s=1} = 2, y'_{01} = 2, z'_{01} = 0$$

$$x'_{02} = 2s_{s=-1} = -2, y'_{02} = 2, z'_{02} = 0$$

$$x'_{03} = 2s_{s=0} = 0, y'_{03} = -1, z'_{03} = 0$$

So, the tangent lines:

$s_{01} = 1$:

$$\frac{x - 0}{2} = \frac{y - 0}{2} = \frac{z - 0}{0}$$

$s_{02} = -1$:

$$\frac{x - 0}{-2} = \frac{y - 0}{2} = \frac{z - 0}{0}$$

$s_{03} = 0$:

$$\frac{x + 1}{0} = \frac{y - 0}{-1} = \frac{z - 0}{0}$$