Answer on question 37118 – Math – Differential Geometry

Let $r(s) = (x(s), y(s), 0)$ be a unit speed curve prove that $K = \frac{|x'y'' - x''y'|}{((x')^2 + (y')^2)^{\frac{3}{2}}}$.

Solution

The curvature can be found using the following formula

$$K = \frac{|r'(s) \times r''(s)|}{|r'(s)^3|}. \qquad (*)$$

Let us apply this formula for our case.

$$|r'(s) \times r''(s)| = \begin{vmatrix} i & j & k \\ x' & y' & 0 \\ x'' & y'' & 0 \end{vmatrix} = |x'y'' - x''y'||k| = |x'y'' - x''y'|.$$

and

$$|r'(s)^3| = ((x')^2 + (y')^2)^{\frac{3}{2}}.$$

Substituting these into (*) we get

$$K = \frac{|x'y'' - x''y'|}{((x')^2 + (y')^2)^{\frac{3}{2}}}.$$

QED.