Taylors expansion of f(x) = Sin x at x = 0.

Solution

$$f(x)=\sin x \Rightarrow f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\cdots+\frac{f^{(n)}(x)}{n!}x^n+\frac{f^{(n+1)}(\theta x)}{(n+1)!}x^{n+1},$$

$$\text{where } \theta \in (0,1);$$

Calculate

$$f(0)=\sin 0=0;$$

$$f'(0)=\cos 0=1;$$

$$f''(0)=-\sin 0=0;$$

$$f'''(0)=-\cos 0=-1;$$

$$f^{(4)}(0)=\sin 0=0;$$

$$f^{(5)}(0)=\cos 0=1;$$

$$f^{(6)}(x)=-\sin x\,;$$

Hence:

$$\sin x=\frac{1}{1!}x+\frac{-1}{3!}x^3+\frac{1}{5!}x^5+\frac{-\sin(\theta x)}{6!}x^6=x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{\sin\theta x}{720}x^6.$$