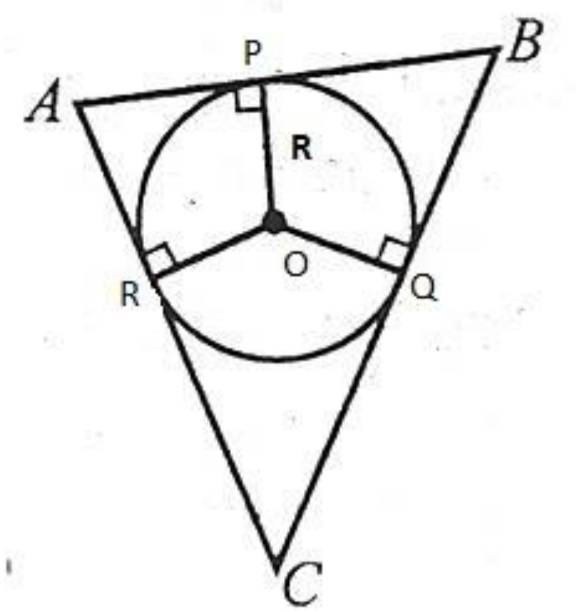
Answer on Question#37115 – Math - Trigonometry

Question.

The sides AB,BC And CA of the triangle ABC touches a circle with centre o and radius R at point P,Q,R ...1.prove that AB+CQ=AC+BQ...2.area of triangle ABC =1/2 perimeter* radius..

Solution.



1. As circle is inscribed of a triangle we can write the next equalities:

$$AR = AP, BP = BQ, CR = CQ. \ (1)$$

$$AB = AP + BP, AC = AR + CR, BC = BQ + CQ. \ (2)$$

So,

$$AB + CQ = (2)AP + BP + CQ \quad (3)$$

$$AC + BQ = (2)AR + CR + BQ \quad (4)$$

According to (1) rewrite the eq. (4):

$$AC + BQ = (2)AR + CR + BQ = (1)AP + CQ + BP \ (5)$$

Thus, we can see that (5) and (3) is the same. So, we proved that $AB + CQ = AC + BQ$.

2. The area of triangle ABC is:

$$S_{ABC} = S_{AOC} + S_{AOB} + S_{BOC}$$

$$S_{AOC} = \frac{1}{2} R * AC$$

$$S_{AOB} = \frac{1}{2} R * AB$$

$$S_{BOC} = \frac{1}{2} R * BC$$

Thus, we have the next equality for area ABC:

$$S_{ABC} = \frac{1}{2} R * AC + \frac{1}{2} R * AB + \frac{1}{2} R * BC = \frac{1}{2} R * (AB + BC + AC) = \frac{1}{2} R * P,$$

where $P -$ perimeter ABC.