Question: When polluted water begins to flow into an unpolluted pond, the concentration of pollutant, c, in the pond at t minutes is modelled by $c(t)=\frac{27t}{10000+3t}$, where c is measured in kilograms per cubic metre. Determine the rate at which the concentration is changing after

a) 1 h
b) one week.

Solution. We need to find the instantaneous rate of change for $c(t)$. First, apply the formula for average rate of change:

$$\frac{c(t+h)-c(t)}{h}=\frac{1}{h}*\left(\frac{27t+27h}{10000+3t+3h}-\frac{27t}{10000+3t}\right)$$

Bring the expression at the right to a common denominator and simplify:

$$\frac{c(t+h)-c(t)}{h}=\frac{1}{h}*\frac{270000h}{(10000+3t+3h)(10000+3t)}=\frac{270000}{(10000+3t+3h)(10000+3t)}.$$

We now find the instantaneous rate of change as the limit of average rate of change as h tends towards zero:

$$\lim_{h\to 0}\frac{c(t+h)-c(t)}{h}=$$
$$=\lim_{h\to 0}\frac{270000}{(10000+3t+3h)(10000+3t)}=\frac{270000}{(10000+3t)(10000+3t)}$$
$$=\frac{270000}{(10000+3t)^2}.$$

Now let us find the rate of change for values of t required in the question:

a) $1\text{ h}=60\text{ min}.\ t=60$
Rate of change:
$$\frac{270000}{(10000+3*60)^2}=\frac{270000}{10180^2}\approx 0.0026\ (\text{kg/m}^3/\text{min})$$
b) $1\text{ week}=7*24\text{ h}=7*24*60\text{ min}=10080\text{ min}.\ t=10080$
Rate of change:
$$\frac{270000}{(10000+3*10080)^2}=\frac{270000}{40240^2}\approx 0.0001667\ (\text{kg/m}^3/\text{min})$$

Answer. The rate of change is:

a) After 1 h – approximately $0.0026\ \text{kg/m}^3/\text{min}$.
b) After one week – approximately $0.00017\ \text{kg/m}^3/\text{min}$.