Answer on question 37101 – Math – Algebra

An economist for a sporting goods company estimates the revenue and cost functions for the production of a new snowboard. These functions for the production of a new snowboard. These functions are R(x) = -x^2 + 10x and C(x) = 4x + 5, respectively, where x is the number of snowboards produced, in thousands. The average profit is defined by the function AP(x) = P(x)/x, where P(x) is the profit function. Determine the production levels that make AP(x) > 0.

Solution

The profit is $P(x) = R(x) - C(x) = -x^2 + 10x - 4x + 5 = -x^2 + 6x + 5.$

Then $AP(x) = \frac{-x^2+6x+5}{x} > 0.$

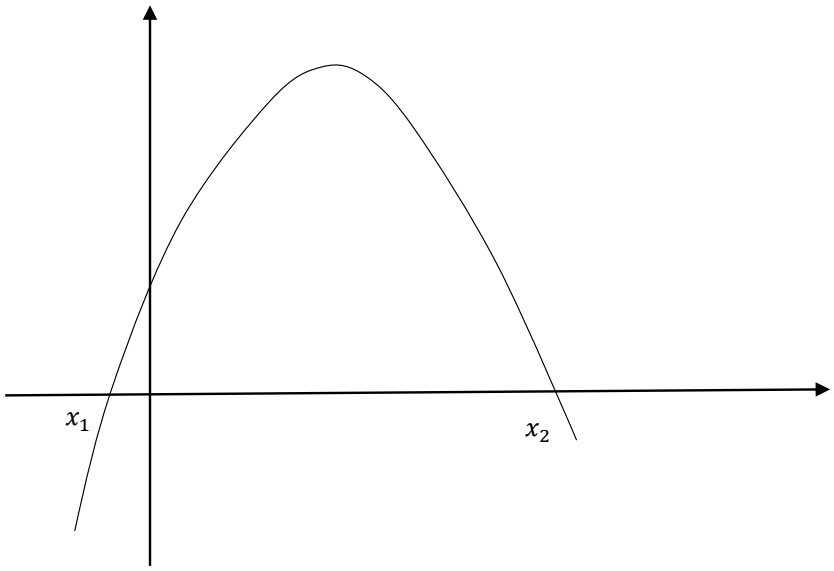
As x>0 then the sign of AP(x) depends on numerator only. We need to solve the following inequality

$$-x^2 + 6x + 5 > 0$$

$$D = 36 + 20 = 56$$

$$x_1 = \frac{-6 - \sqrt{56}}{-2} \approx 6.7417, \qquad x_2 = \frac{-6 + \sqrt{56}}{-2} \approx -0.7417$$

The sketch of this parabola is



Therefrom, AP(x) > 0 if $0 < x < 6.7417.$

Answer: The average profit is positive if the number of snowboards less than 67418.