Answer on Question#37090 – Math- Other

Let $I_C(x)$ be the indicator function of the closed convex set C. Show that the sub-differential of the function I_C at a point c in C is the normal cone to C at the point c.

Solution.

We have the convex set C and the indicator function $I_C(x)$. We know that:

1. For $c \notin C$, $\partial I_C(c) = \emptyset$ (by convention).
2. For $c \in C$, we have $g \in \partial I_C(c)$ if $I_C(z) \geq I_C(c) + g'(z-c), \forall\, z \in C$, or equivalently $g'(z-c) \leq 0$ for all $z \in C$.

Thus $I_C(c)$ is the normal cone of C at c, denoted $N_C(c)$:

$N_C(c) = \{g \mid g'(z-c) \leq 0, \forall\, z \in C\}$.