Answer on question 37071 – Math – Statistics and Probability

Suppose there are N = 10 urns behind a curtain, such that you cannot see them. The urns are numbered from i = 1,…, 10. Urn i contains ten balls: i white balls and 10 - i red balls. A person behind the curtain picks one urn at random (all urns are equiprobable), picks ten balls with replacement from this urn and notes the result. Afterwards, the person tells you that NW = 3 white balls were drawn (and accordingly 10 - 3 = 7 red balls). What is the probability that the person has chosen urn i ∈ {1, 2,…,10}? Please give a derivation and numbers.

Solution.

Let H_i is an event that a person choose the urn number i. Then $P(H_i) = \frac{1}{10}$, for every $i = 1, \dots, 10$ and A is an event that a person choose 3 white balls.

Let us find $P(A|H_i) = C_{10}^3 \left(\frac{i}{10}\right)^3 \left(\frac{10-i}{10}\right)^7 = \frac{12i^3(10-i)^7}{10^9}$, for every $i = 1, \dots, 10$.

Using Bayes' Theorem we get.

$$P(H_i|A) = \frac{P(A|H_i)P(H_i)}{\sum_{i=1}^{10} P(A|H_i)P(H_i)} = \frac{\frac{12i^3(10-i)^7}{10^9} * \frac{1}{10}}{\sum_{i=1}^{10} \frac{12i^3(10-i)^7}{10^9}} = \frac{i^3(10-i)^7}{10\sum_{i=1}^{10} i^3(10-i)^7}.$$

Find the values for some i.

If i=1 than the probability that the person has chosen the first urn is

$$P(H_1|A) = \frac{9^7}{10\sum_{i=1}^{10} i^3(10-i)^7} = \frac{9^7}{10(9^7 + 2^3 8^7 + 3^3 7^7 + \cdots + 9^3)} \approx 0.0063;$$

For i=2:

$$P(H_2|A) = \frac{2^3 8^7}{10\sum_{i=1}^{10} i^3(10-i)^7} = \frac{2^3 8^7}{10(9^7 + 2^3 8^7 + 3^3 7^7 + \cdots + 9^3)} \approx 0.0221;$$

And so on.

Answer: the probability that the person has chosen urn i is $\frac{i^3(10-i)^7}{10\sum_{i=1}^{10} i^3(10-i)^7}$.