Answer on Question #37040 – Math - Calculus

Assignment

the temperature of a vase is 1,080 degrees F to begin with, and the temperature of the room with the cooling table it is placed on is 80 degrees F. 4 minutes later, the temperature of the vase is 830 degrees F. Find a formula for the temperature of the vase after t minutes.

Solution

We use Newton's law of cooling: the rate at which an object cools (or warms up, if it's cooler than Its surroundings) is proportional to the difference between its temperature and that of its surroundings. In our case the temperature of the room is 80 degrees, so Newton`s law of cooling states that $Q^{'}(t)$ is proportional to $Q - 80$, the difference between the temperature of the vase and the room. In symbols, we have $Q^{'} = -k(Q - 80)$, where k is some positive constant. We know the initial temperature of the vase is $Q(0) = 1080$. We need to solve the problem

$Q^{'} = -k(Q - 80)$, $Q(0) = 1080$, $Q(4) = 830$.

Rewrite differential equation as

$Q^{'} + kQ = 80k$. (1)

Equation (1) is a first-order non-homogeneous linear differential equation.

$Q^{'} + kQ = 0$ (2)

Equation (2) is a first-order homogeneous linear differential equation.

The general solution of (2) is $Q(t) = Ce^{-kt}$, where C is some constant.

We check that $Q = 80$ is a particular solution of (1):

$(80)^{'} + 80k = 80k$, $80k = 80k$ – it is true.

The general solution of (1) is the sum of general solution of (2) and a particular solution of (1), i.e.

$Q(t) = Ce^{-kt} + 80$, (3)

where C is some constant. To find the unknown C and k, we apply conditions $Q(0) = 1080$,

$Q(4) = 830$ to (3).

So, $Q(0) = Ce^{-k*0} + 80 = C + 80 = 1080$, where from we conclude

$C = 1080 - 80 = 1000$. (4)

Further, $Q(4) = Ce^{-4k} + 80 = 1000e^{-4k} + 80 = 830$, where from we conclude $1000e^{-4k} = 750$, divide by 1000 and come to $e^{-4k} = 0.75$. Take natural logs of both sides, "bring down" the power: $-4k * \ln(e) = \ln(0.75)$. We know $\ln(e) = 1$ and solve for k :

$k = -\frac{\ln(0.75)}{4}$. (5)

Taking into account (4) and (5), formula (3) becomes

$Q(t) = 1000 * e^{\frac{\ln(0.75)}{4}t} + 80$ or

$Q(t) = 1000 * (0.75)^{\frac{t}{4}} + 80$ (degrees F)

Answer: $1000 * (0.75)^{\frac{t}{4}} + 80$ (degrees F).