Answer on question 36977 – Math – Discrete Mathematics

How many solutions are there to the equation x1+x2+x3+x4+x5=21, where xi, i=1,2,3,4,5, is a nonnegative integer such that

a) $x1 \geq 1$?

b) $x_i \geq= 2, for\ i = 1,2,3,4,5$?

c) $0 \leq x1 \leq 10$?

d) $0 \leq x1 \leq 3, 1 \leq x2 < 4$, and $x3 \geq 15$?

Solution

a) Let $y_1 = x_1 - 1, y_2 = x_2, y_3 = x_3, y_4 = x_4, y_5 = x_5$. Substitute these into our equation

$$y_1 + y_2 + y_3 + y_4 + y_5 = 20$$

And $y_i \geq 0$ for i=1, 2, 3, 4, 5.

There is a 1-1 correspondence between the solutions and reorderings of 20 ones and 4 zeros (y_1 is the number of ones before the first zero, $y_2$the number of ones between the first and the second zero, and so on).

Hence the answer is

$$C_{20+4}^4 = C_{24}^4 = 10626.$$

b) Let $y_1 = x_1 - 2, y_2 = x_2 - 2, y_3 = x_3 - 2, y_4 = x_4 - 2, y_5 = x_5 - 2$. Substitute these into our equation

$$y_1 + y_2 + y_3 + y_4 + y_5 = 21 - 10 = 11$$

Similarly to the previous case we get

$$C_{11+4}^4 = C_{15}^4 = 1365.$$

c) Using the sum rule, the number of solutions with $0 \leq x1 \leq 10$ added to the number of solutions with $x_1 \geq 11$ gives all non-negative integer solutions. Thus the number of solutions with $0 \leq x1 \leq 10\ is$

$$C_{25}^4 - C_{14}^4 = 12650 - 1001 = 11649.$$

d) Let $y_1 = x_1, y_2 = x_2 - 1, y_3 = x_3 - 15, y_4 = x_4, y_5 = x_5$. Substitute these into our equation

$$y_1 + y_2 + y_3 + y_4 + y_5 = 21 - 1 - 15 = 5,$$

where $y_1 \leq 3, y_2 \leq 2, y_i \geq 0, i = 3, 4, 5$. Similarly to the previous case we get

$$C_9^4 - C_6^4 - C_7^4 = 76.$$

Answer: a) 10626; b) 1365; c) 11649; d) 76.