Answer on Question #36949 – Math – Statistics and Probability

A survey of adults ages 18-29 found that 87% use the internet. You randomly select 100 adults ages 18-29 and ask them if they use the internet.

a) find the probability that exactly 89 people say they use the internet

b) find the probability that at least 89 people say they use the internet

c)Find the probability that fewer than 89 people say they use the internet

d) are any of the probabilities in parts a-c unusual? Explain!

Solution

Designating each reply by S and assuming that the results for individual adults are independent, we try the binomial distribution with $n = 100$ and $= .87$, $q = 1 - p = 1 - (.87) = .13$ for X=number of adults who use internet.

When np and $n(1-p)$ are both large, say, greater than 15, the binomial distribution is well approximated by the normal distribution having mean $= np = 100 * (.87) = 87$ and standard deviation $sd = \sqrt{np(1-p)} = \sqrt{100 * (.87) * (1 - (.87))} \approx 3.363$.

The standardized variable is $Z = \frac{X - mean}{sd} = \frac{X-87}{3.363}$.

The binomial probability of $[a \le X \le b]$ is approximated by the normal probability of $[a - \frac{1}{2} \le X \le b + \frac{1}{2}]$.

The event $[a \le X \le b]$ includes both endpoints. The appropriate continuity correction is to subtract $\frac{1}{2}$ from the lower end and add $\frac{1}{2}$ to the upper end.

a) Pr[X=k] $=\binom{n}{k} * p^k * (1-p)^{n-k}$

Pr[X=89]=$\binom{100}{89} * (.87)^{89} * (.13)^{11} \approx 0.103$

The normal approximation to the binomial gives

Pr[exactly 89]= Pr[X=89]≈ $\Pr\left[89 - \frac{1}{2} \le X \le 89 + \frac{1}{2}\right] = \Pr[88.5 \le X \le 89.5] =$

$= \Pr\left[\frac{88;.5 - 87}{3.363} \le \frac{X - 87}{3.363} \le \frac{89.5 - 87}{3.363}\right] \approx \Pr[0.446 \le Z \le 0.743] =$

$= \Pr[Z \le 0.743] - \Pr[Z \le 0.446] \approx 0.7703 - 0.6718 = 0.0985$

Probabilities $\Pr[Z \le 0.743]$ and $\Pr[Z \le 0.446]$ are found from normal tables.

b) First we find probability for $[X > 89]$, we reason that 89 is not included so that $[X \ge 89.5]$ is the event of interest:

$\Pr[X \ge 89.5] = \Pr\left[\frac{X-87}{3.363} \ge \frac{89.5-87}{3.363}\right] \approx \Pr[Z \ge 0.743] = 1 - .7703 = 0.2297.$

Finally, we find

$\Pr[at\ least\ 89] = |additivity| = \Pr[X = 89] + \Pr[X > 89] \approx 0.0985 + 0.2297 =$
=0.3282.

c) $\Pr[\text{fewer than } 89] = \Pr[X < 89] = |complementary\ event| = 1 - \Pr[X \ge 89] =$

$= 1 - \Pr[X = 89] - \Pr[X > 89] \approx 1 - 0.3282 = 0.6718$

d) Probability in a) seems unusual because it is difficult to indicate the exact number of internet users, we can speak about this value approximately (is greater than, is less than, ranges from the lower end to the upper end).