A cone has a radius of 4 and a height of 8. The smaller cone is 1/8 of the bigger cones volume. What are the dimensions of the smaller cone?

Solution:

We have

$$V_b = \frac{1}{3}\pi R_b^2 h_b$$

where V_b, R_b, h_b is the volume, radius and height of the bigger cone. Because $R_b = 4$ and $h_b = 8$ then

$$\frac{h_b}{R_b} = \frac{8}{4} = 2 \Rightarrow h_b = 2R_b.$$

Thus

$$V_b = \frac{1}{3}\pi R_b^2 \cdot 2R_b,$$

$$V_b = \frac{2}{3}\pi R_b^3.$$

Denote V_s, R_s, h_s is the volume, radius and height of the smaller cone. So

$$h_s = 2R_s, V_s = \frac{2}{3}\pi R_s^3.$$

Thus we have

$$\frac{V_s}{V_b} = \frac{\frac{2}{3}\pi R_s^3}{\frac{2}{3}\pi R_b^3} = \frac{1}{8},$$

$$\frac{R_s^3}{R_b^3} = \frac{1}{8},$$

$$\left(\frac{R_s}{R_b}\right)^3 = \left(\frac{1}{2}\right)^3,$$

$$\frac{R_s}{R_b} = \frac{1}{2},$$

$$R_s = \frac{1}{2}R_b = \frac{1}{2}\cdot 4 = 2.$$

Also

$$h_s = 2R_s = 4.$$

Answer: $R_s = 2, h_s = 4$.