Question#36856 – Math - Algebra

$$p^2 = qy + rz;$$

$$q^2 = px + rz;$$

$$r^2 = px + qy;$$

$$\frac{x}{p+x} + \frac{y}{q+y} + \frac{z}{r+z} = ?$$

Solution.

$$p^2 + q^2 + r^2 = (qy + rz) + (px + rz) + (px + qy) = 2(px + qy + rz);$$

$$-p^2 + q^2 + r^2 = -(qy + rz) + (px + rz) + (px + qy) = 2px;$$

$$p^2 - q^2 + r^2 = (qy + rz) - (px + rz) + (px + qy) = 2qy;$$

$$p^2 + q^2 - r^2 = (qy + rz) + (px + rz) - (px + qy) = 2rz;$$

Consider the case, when one of the numbers p, q, r is equal to 0. Without loss of generality, $p = 0$:

$$\begin{cases} qy + rz = 0 \\ q^2 = rz \\ r^2 = qy \end{cases} \Rightarrow q^2 + r^2 = 0 \Rightarrow p = q = r = 0 \Rightarrow \frac{x}{p+x} + \frac{y}{q+y} + \frac{z}{r+z} = 3;$$

Now assume that $p, q, r \neq 0$.

Hence:

$$\frac{x}{p+x} = \frac{2px}{2(p^2 + px)} = \frac{-p^2 + q^2 + r^2}{2(px + qy + rz)} = \frac{-p^2 + q^2 + r^2}{p^2 + q^2 + r^2};$$

$$\frac{y}{q+y} = \frac{2qy}{2(q^2 + qy)} = \frac{p^2 - q^2 + r^2}{2(px + qy + rz)} = \frac{p^2 - q^2 + r^2}{p^2 + q^2 + r^2};$$

$$\frac{z}{r+z} = \frac{2rz}{2(r^2 + rz)} = \frac{p^2 + q^2 - r^2}{2(px + qy + rz)} = \frac{p^2 + q^2 - r^2}{p^2 + q^2 + r^2};$$

$$\frac{x}{p+x} + \frac{y}{q+y} + \frac{z}{r+z} = \frac{-p^2 + q^2 + r^2}{p^2 + q^2 + r^2} + \frac{p^2 - q^2 + r^2}{p^2 + q^2 + r^2} + \frac{p^2 + q^2 - r^2}{p^2 + q^2 + r^2} =$$

$$= \frac{p^2 + q^2 + r^2}{p^2 + q^2 + r^2} = 1.$$

Answer.

$$\frac{x}{p+x} + \frac{y}{q+y} + \frac{z}{r+z} = \begin{cases} 1, \text{if } p, q, r \neq 0. \\ 3, \text{if } p = q = r = 0. \end{cases}$$