Calculate the left Riemann sum for $f(x) = \frac{1}{\sqrt{x}}$ over [1, 4] with n = 6(6 rectangles). When rounding, round your answers to four decimal places.

Solution:

We have

$$\int_1^4 \frac{1}{\sqrt{x}}dx \approx \sum_{i=0}^{5} h\frac{1}{\sqrt{x_i}} = h\sum_{i=0}^{5}\frac{1}{\sqrt{x_i}}$$

where $h = \frac{4-1}{n} = \frac{3}{6} = \frac{1}{2}$; $x_i = 1 + i \cdot h = 1 + \frac{i}{2}$. So

$$\int_1^4 \frac{1}{\sqrt{x}}dx \approx h\sum_{i=0}^{5}\frac{1}{\sqrt{x_i}} = \frac{1}{2}\left(\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{1+1\cdot\frac{1}{2}}} + \frac{1}{\sqrt{1+2\cdot\frac{1}{2}}} + \frac{1}{\sqrt{1+3\cdot\frac{1}{2}}} + \frac{1}{\sqrt{1+4\cdot\frac{1}{2}}} + \right.$$

$$\left. + \frac{1}{\sqrt{1+5\cdot\frac{1}{2}}}\right) = \frac{1}{2}\left(1 + \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{2}} + \sqrt{\frac{2}{5}} + \frac{1}{\sqrt{3}}\right) \approx 1.8667$$

The exact solution is

$$\int_1^4 \frac{1}{\sqrt{x}}dx = 2\sqrt{x}\Big|_1^4 = 2\left(\sqrt{4} - \sqrt{1}\right) = 2(2-1) = 2.$$

Answer:

$$\approx 1.8667$$