Question #36767, Calculus

a) Find the critical points, if any, of the following function on the given interval.

b) Determine the absolute extreme values of f on the given interval.

c) Use a graphing utility to confirm your conclusions.

f(x)=sin(3x)on [−π4,π3]

I Know f′(x)=3cos(3x).

However, I'm not exactly sure how to determine the zeros for x for a trig function.

and even I know how to find the zeros of a cosine function I don't know what to do with them after that.

please if somebody can solve this I've been at it for over 5 hr's

Solution

1. f(x)=sin(3x) on [−π4,π3]

a) To determine the critical value(s) we must first find $f'(x)$:

$f'(x) = 3\cos(3x)$.

Next we must find value(s) of c such that $f'(c) = 0$.

So, $f'(x) = 0 \Leftrightarrow 3\cos(3x) = 0 \Leftrightarrow \cos(3x) = 0$.

The last equation has the general solution $3x = \mathrm{Cos}^{-1}(0) + \pi n = \pi/2 + \pi n$, where n is an integer (including zero). Hence, $x = \pi/6 + \pi n/3$, where n is an integer.

For −π4 ≤x≤ π3 we have $-12 \le n \le 8$, i. e., 21 critical points for given interval.

b) By setting $n = 2k$ we get $x = \pi/6 + 2\pi k/3$, for $-6 \le k \le 4$ or $x = -23\pi/6, -19\pi/6, -5\pi/2, -11\pi/6, -7\pi/6, -\pi/2, \pi/6, 5\pi/6, 3\pi/2, 13\pi/6, 17\pi/6$.

And then we find the function values at these critical points

$f(\pi/6 + 2\pi k/3) = \sin(\pi/2 + 2\pi k) = 1$.

By setting $n = 2k-1$ we obtain $x = -\pi/6 + 2\pi k/3$, for $-5 \le k \le 4$ or $x = -21\pi/6, -17\pi/6, -13\pi/6, -3\pi/2, -5\pi/6, -\pi/6, \pi/2, 7\pi/6, 11\pi/6, 15\pi/6$.

For these critical points we get

$f(-\pi/6 + 2\pi k/3) = \sin(-\pi/2 + 2\pi k) = -1$.

The values of the function at the end points of interval are

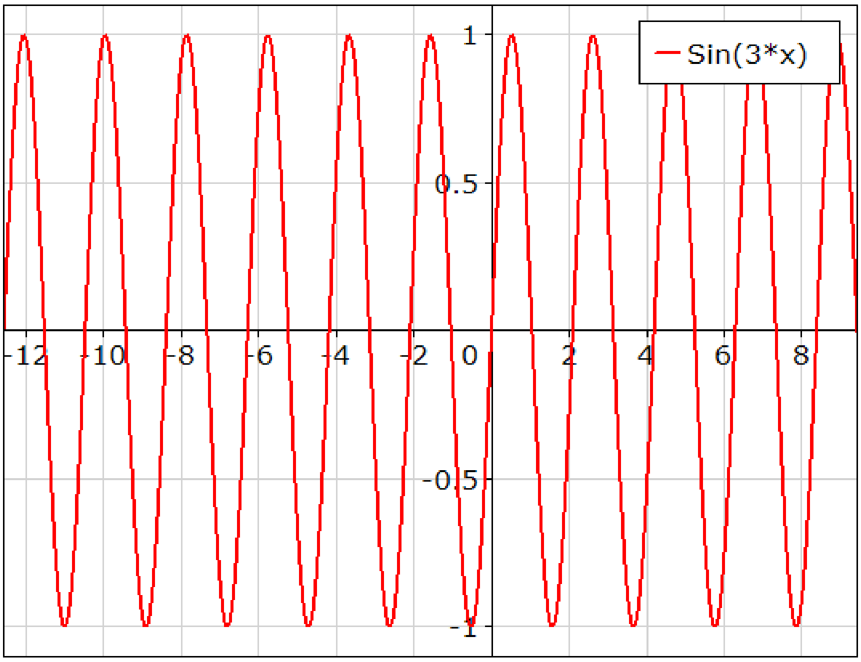
$f(-\pi 4) = \sin(-12\pi) = 0$, $f(\pi 3) = \sin(9\pi) = 0$.

Answers

a) The critical points are $x = -\pi/6 + 2\pi k/3$, for $-5 \le k \le 4$ and $x = \pi/6 + 2\pi k/3$, for $-6 \le k \le 4$.

b) The absolute minimum occurs at the critical values x = $-\pi/6 + 2\pi k/3$, for $-5 \le k \le 4$ and is equal to -1; the absolute maximum occurs at the critical values x = $\pi/6 + 2\pi k/3$, for $-6 \le k \le 4$ and is equal to 1.

c) We use SpeQ 3.4 graphing utility to confirm our conclusions.



2. f(x)=sin(3x) on $[-\pi/4, \pi/3]$

a) To determine the critical value(s) we must first find $f'(x)$:

$f'(x) = 3\cos(3x)$.

Next we must find value(s) of c such that $f'(c) = 0$.

So, $f'(x) = 0 \Leftrightarrow 3\cos(3x) = 0 \Leftrightarrow \cos(3x) = 0$.

The last equation has the general solution $3x = \mathrm{Cos}^{-1}(0) + \pi n = \pi/2 + \pi n$, where n is an integer (including zero). Hence, $x = \pi/6 + \pi n/3$, where n is an integer.

For $-\pi/4 \le x \le \pi/3$ we have $n = -1, 0$, i. e., two critical points for the given interval x = $\pm\pi/6$.

b) First, we find the function values at these critical points

$f(\pi/6) = \sin(\pi/2) = 1$,

$f(-\pi/6) = \sin(-\pi/2) = -1$.

The values of the function at the end points of interval are

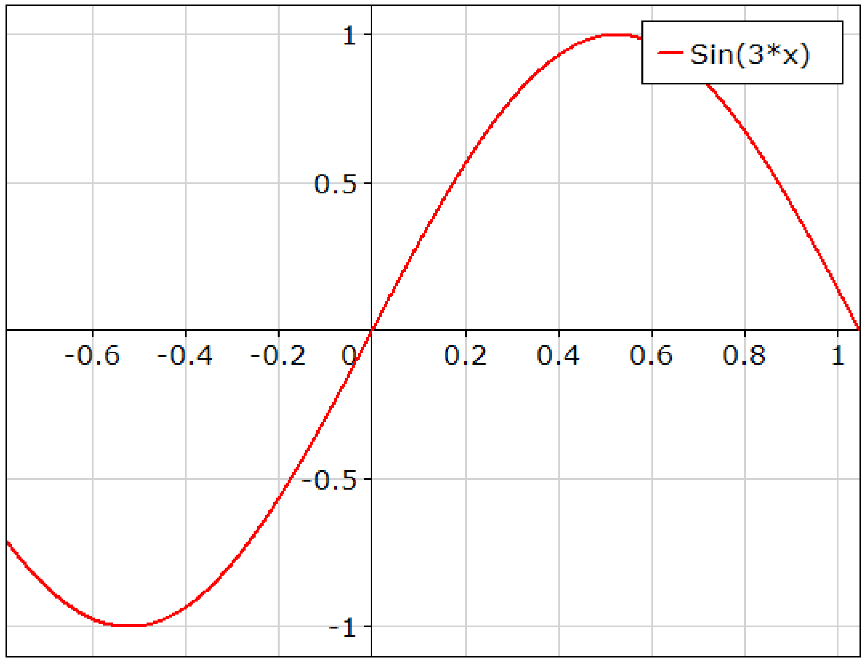
$f(-\pi/4) = \sin(-3\pi/4) = -\frac{\sqrt{2}}{2}$, $f(\pi/3) = \sin(\pi) = 0$.

Answers

a) The critical points are x = $-\pi/6$ and x = $\pi/6$.

b) The absolute minimum occurs at the critical value x = $-\pi/6$ and is equal to -1 ; the absolute maximum occurs at the critical value $\pi/6$ and is equal to 1.

c) We use SpeQ 3.4 graphing utility to confirm our conclusions.


Sin(3*x)
1
0.5
-0.5
-1
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1