We have the equation

$$4x^5 + 4x^4 + 35x^3 + 35x^2 - 9x - 9 = 0$$

This is the polynomial of the 5-th order with leading coefficient 4 and free coefficient 9.

Solution

Method 1.

We know that all the rational roots of the polynomial are in the form $\frac{p}{q}$, where p is a divisor of free coefficient and q is a divisor of a leading coefficient.

Divisors of -9: 1,-1,3,-3,9,-9

Divisors of 4: 1,-1,2,-2,4,-4

Thus we have such possible variants:

$$p = 1; -1; 3; -3; 9; -9$$

$$q = 1; -1; 2; -2; 4; -4$$

Substituting all the possible combinations of $\frac{p}{q}$ into the equation e get that rational solutions are

$$x_1 = -1$$

$$x_2 = -\frac{1}{2}$$

$$x_3 = \frac{1}{2}$$

Method 2.

We need to find rational solutions. Rewrite the equation as follows:

$4x^4(x + 1) + 35x^2(x + 1) - 9(x + 1) = 0$;

Multiplier $(x + 1)$ is common for all the terms:

$$(x + 1)(4x^4 + 35x^2 - 9) = 0$$

We have the union of solutions of the following equations:

$x + 1 = 0$ or $4x^4 + 35x^2 - 9 = 0$

$x = -1$ or $4x^4 + 35x^2 - 9 = 0$

To solve equation $4x^4 + 35x^2 - 9 = 0$ we substitute $x^2 := t \geq 0$ and get a square equation

$4t^2 + 35t - 9 = 0$. Discriminant $D = 35^2 + 4 * 4 * 9 = 1369$ and roots are

$t_{1;2} = \frac{-35 \pm 37}{8} = \frac{-72}{8}; \frac{2}{8} = -9; \frac{1}{4}$. But $t_1 = -9$ does not satisfy because $t \geq 0$, that is why $t = t_2 = \frac{1}{4}$; or $x^2 = \frac{1}{4}$, whence $x = \pm\frac{1}{2}$.

ANSWER: $-1; -\frac{1}{2}; \frac{1}{2}$