

Task

Given that P(h,k) is a point equidistant from the points A(3,5) and B(7,-1), prove that $3k-2h+4=0$

Solution

As it is given, $PA = PB$, so we can find squares of lengths and then equate them:

$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ - general formula of the square of the distance between points A ($x_1; y_1$) and B($x_2; y_2$).

In this case:

$$PA^2 = (h - 3)^2 + (k - 5)^2$$

$$PB^2 = (h - 7)^2 + (k + 1)^2$$

Since $PA = PB$, so $PA^2 = PB^2$;

$$(h - 7)^2 + (k + 1)^2 = (h - 3)^2 + (k - 5)^2$$

$$h^2 - 6h + 9 + k^2 - 10k + 25 = h^2 - 14h + 49 + k^2 + 2k + 1$$

$$12k - 8h + 16 = 0 \quad /:4$$

$$3k - 2h + 4 = 0$$

Answer

Proved: $3k - 2h + 4 = 0$