The manager of a large apartment complex knows from experience that 120 units will be occupied if the rent is 500 dollars per month. A market survey suggests that, on average, one additional unit will remain vacant for each 7 dollar increase in rent. Similarly, one additional unit will be occupied for each 7 dollar decrease in rent.

Let the rent on an apartment be x dollars per month, and let N be the number of apartments rented each month, and let R be the revenue (the gross income) brought in each month by the apartment manager. Find $N(x)$ and $R(x)$.

Solution.

Let's find $N(x)$. We have linear equation for $N(x)$:

$$N(x) = mx + b$$

$$N(500) = 120$$

$$N(507) = 119$$

Then

$$m = \frac{119 - 120}{507 - 500} = -1/7$$

$$N = -\frac{1}{7}x + b$$

Find b by putting in the coordinate pair $(500, 120)$:

$$120 = -\frac{1}{7} \cdot 500 + b$$

$$b = 120 + \frac{500}{7} = 191.43$$

So

$$N(x) = -\frac{1}{7}x + 191.43$$

$$R(x) = N(x) \cdot x = -\frac{1}{7}x^2 + 191.43x$$

Answer:

$$N(x) = -\frac{1}{7}x + 191.43$$

$$R(x) = -\frac{1}{7}x^2 + 191.43x$$