Solve the system of differential equations dy/dx =xz+1, dz/dx =-xy for x=0.3 using 4th order R-K method with y(0)=0, z(0)=1.

Solution:

Suppose we have next problem

$$\frac{dy}{dx} = f(z,x), \quad y(x_0) = y_0.$$

Then steps of 4th order R-K method are next

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + k_2 + k_3 + k_4),$$

$$x_{n+1} = x_n + h,$$

where

$$k_1 = f(z_n, x_n),$$

$$k_2 = f\left(z_n + \frac{1}{2}h, x_n + \frac{h}{2}k_1\right),$$

$$k_3 = f\left(z_n + \frac{1}{2}h, x_n + \frac{h}{2}k_2\right),$$

$$k_4 = f(z_n + h, x_n + hk_3).$$

In our case $x_0 = 0$, $y_0 = 0$, $z_0 = 1$, $h = 0.3$. Then we have

$$y_1 = y_0 + \frac{1}{6}h\left(k_1^{(y)} + k_2^{(y)} + k_3^{(y)} + k_4^{(y)}\right);$$

$$z_1 = z_0 + \frac{1}{6}h\left(k_1^{(z)} + k_2^{(z)} + k_3^{(z)} + k_4^{(z)}\right);$$

$$k_1^{(y)} = f^{(y)}(z_0, x_0) = x_0 z_0 + 1 = 1;$$

$$k_1^{(z)} = f^{(z)}(y_0, x_0) = -x_0 y_0 = 0;$$

$$k_2^{(y)} = f^{(y)}\left(z_0 + \frac{1}{2}h, x_0 + \frac{h}{2}k_1^{(y)}\right) = \left(x_0 + \frac{h}{2}k_1^{(y)}\right)\left(z_0 + \frac{1}{2}h\right) + 1 =$$

$$= \left(0 + \frac{0.3}{2}\right)\left(1 + \frac{0.3}{2}\right) + 1 = 1.1725;$$

$$k_2^{(z)} = f^{(z)}\left(y_0 + \frac{1}{2}h, x_0 + \frac{h}{2}k_1^{(z)}\right) = -\left(x_0 + \frac{h}{2}k_1^{(z)}\right)\left(y_0 + \frac{1}{2}h\right) =$$

$$= -(0+0)\left(0 + \frac{0.3}{2}\right) = 0;$$

$$k_3^{(y)} = f^{(y)}\left(z_0 + \frac{1}{2}h, x_0 + \frac{h}{2}k_2^{(y)}\right) = \left(x_0 + \frac{h}{2}k_2^{(y)}\right)\left(z_0 + \frac{1}{2}h\right) + 1 =$$

$$= \left(0 + \frac{0.3}{2} \cdot 1.1725\right)\left(1 + \frac{0.3}{2}\right) + 1 = 1.20225625;$$

$$k_3^{(z)} = f^{(z)}\left(y_0 + \frac{1}{2}h, x_0 + \frac{h}{2}k_2^{(z)}\right) = -\left(x_0 + \frac{h}{2}k_2^{(z)}\right)\left(y_0 + \frac{1}{2}h\right) =$$

$$= -(0+0)\left(0 + \frac{0.3}{2}\right) = 0;$$

$$k_4^{(y)} = f^{(y)}\left(z_0 + h, x_0 + hk_3^{(y)}\right) = (z_0 + h)\left(x_0 + hk_3^{(y)}\right) + 1 =$$

$$= (1 + 0.3)(0 + 0.3 \cdot 1.20225625) + 1 = 1.468879938;$$

$$k_4^{(z)} = f^{(z)}\left(y_0 + h, x_0 + hk_3^{(z)}\right) = -(y_0 + h)\left(x_0 + hk_3^{(z)}\right) =$$

$$= -(0 + 0.3)(0 + 0.3 \cdot 0) = 0.$$

Thus we have

$$y_1 = y_0 + \frac{1}{6}h\left(k_1^{(y)} + k_2^{(y)} + k_3^{(y)} + k_4^{(y)}\right) =$$

$$= 0 + \frac{0.3}{6}(1 + 1.1725 + 1.20225625 + 1.468879938) = 0.242181809;$$

$$z_1 = z_0 + \frac{1}{6}h\left(k_1^{(z)} + k_2^{(z)} + k_3^{(z)} + k_4^{(z)}\right) = 1 + \frac{0.3}{6} \times$$

$$\times (0 + 0 + 0 + 0) = 1.$$

Answer:

$$y|_{x=0.3} = 0.242181809$$

$$z|_{x=0.3} = 1$$